



ETC3550/ETC5550 Applied forecasting

Week 6: Exponential smoothing





Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a "level", "trend" (slope) and "seasonal" component to describe a time series.
- The rate of change of the components are controlled by "smoothing parameters": α, β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

$\begin{array}{ccc} \textbf{General notation} & \texttt{ETS}: \texttt{ExponenTial Smoothing} \\ \nearrow \uparrow \nwarrow \\ \textbf{Error Trend Season} \end{array}$

Error: Additive ("A") or multiplicative ("M")

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Trend: None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

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Error: Additive ("A") or multiplicative ("M")

Trend: None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

Seasonality: None ("N"), additive ("A") or multiplicative ("M")

Additive Error		Seasonal Component			
	Trend	N	А	Μ	
	Component	(None)	(Additive)	(Multiplicative)	
Ν	(None)	A,N,N	A,N,A	A,N,M	
А	(Additive)	A,A,N	A,A,A	A,A,M	
A_d	(Additive damped)	A,A _d ,N	A,A _d ,A	A,A _d ,M	

Multiplicative Error		Seasonal Component			
Trend		Ν	А	Μ	
	Component	(None)	(Additive)	(Multiplicative)	
Ν	(None)	M,N,N	M,N,A	M,N,M	
Α	(Additive)	M,A,N	M,A,A	M,A,M	
A_d	(Additive damped)	M,A _d ,N	M,A _d ,A	M,A _d ,M	

Additive Error		Seasonal Component			
	Trend	N	А	Μ	
	Component	(None)	(Additive)	(Multiplicative)	
Ν	(None)	A,N,N	A,N,A	Δ N M	
А	(Additive)	A,A,N	A,A,A	Λ,Λ,M	
A_d	(Additive damped)	A,A _d ,N	A,A _d ,A	$\Delta, \Delta_{\rm d}, M$	

Multiplicative Error		Seasonal Component			
	Trend	N	А	Μ	
	Component	(None)	(Additive)	(Multiplicative)	
Ν	(None)	M,N,N	M,N,A	M,N,M	
А	(Additive)	M,A,N	M,A,A	M,A,M	
A_d	(Additive damped)	M,A _d ,N	M,A _d ,A	M,A _d ,M	

AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.

From Hyndman et al. (IJF, 2002):

- Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE (or some other criterion).
- Select best method using AICc:
- Produce forecasts using best method.
- Obtain forecast intervals using underlying state space model.

Method performed very well in M3 competition.

Residuals

Response residuals

$$\hat{e}_t$$
 = $y_t - \hat{y}_{t|t-1}$

Innovation residuals

Additive error model:

$$\widehat{arepsilon}_t$$
 = $y_t - \widehat{y}_{t|t-1}$

Multiplicative error model:

$$\hat{\varepsilon}_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}}$$

Your turn

1 Try forecasting the Chinese GDP from the global_economy data set using an ETS model.

Experiment with the various options in the ETS() function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each is doing to the forecasts.

[Hint: use h=20 when forecasting, so you can clearly see the differences between the various options when plotting the forecasts.]

Your turn

- Find an ETS model for the Gas data from aus_production and forecast the next few years.
 - Why is multiplicative seasonality necessary here?
 - Experiment with making the trend damped. Does it improve the forecasts?