MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550 Applied forecasting

Week 6: Exponential smoothing

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Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a "level", "trend" (slope) and "seasonal" component to describe a time series.
- The rate of change of the components are controlled by "smoothing parameters": α, β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

$\nearrow$ $\uparrow$ $\nwarrow$

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

Seasonality: None (`"N"`), additive (`"A"`) or multiplicative (`"M"`)

ETS models

Additive Error

Trend Component		**Seasonal Component** N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	A,N,M
A	(Additive)	A,A,N	A,A,A	A,A,M
A_d	(Additive damped)	A,A_d,N	A,A_d,A	A,A_d,M

Multiplicative Error

Trend Component		**Seasonal Component** N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A_d	(Additive damped)	M,A_d,N	M,A_d,A	M,A_d,M

ETS models

Additive Error

Trend Component		Seasonal Component: N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	~~A,N,M~~
A	(Additive)	A,A,N	A,A,A	~~A,A,M~~
A_d	(Additive damped)	A,A_d,N	A,A_d,A	~~A,A_d,M~~

Multiplicative Error

Trend Component		Seasonal Component: N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A_d	(Additive damped)	M,A_d,N	M,A_d,A	M,A_d,M

AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.

Automatic forecasting

From Hyndman et al. (IJF, 2002):

- Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE (or some other criterion).
- Select best method using AICc:
- Produce forecasts using best method.
- Obtain forecast intervals using underlying state space model.

Method performed very well in M3 competition.

Residuals

Response residuals

$$\hat{e}_t = y_t - \hat{y}_{t|t-1}$$

Innovation residuals

Additive error model:

$$\hat{\varepsilon}_t = y_t - \hat{y}_{t|t-1}$$

Multiplicative error model:

$$\hat{\varepsilon}_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}}$$

Your turn

1. Try forecasting the Chinese GDP from the `global_economy` data set using an ETS model.

   Experiment with the various options in the `ETS()` function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each is doing to the forecasts.

   [Hint: use `h=20` when forecasting, so you can clearly see the differences between the various options when plotting the forecasts.]

Your turn

2. Find an ETS model for the Gas data from `aus_production` and forecast the next few years.
    - Why is multiplicative seasonality necessary here?
    - Experiment with making the trend damped. Does it improve the forecasts?