MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550 Applied forecasting

Week 10: Regression models

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Multiple regression and forecasting

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t.$$

- y_t is the variable we want to predict: the "response" variable
- Each $x_{j,t}$ is numerical and is called a "predictor". They are usually assumed to be known for all past and future times.
- The coefficients $\beta_1, \dots, \beta_k$ measure the effect of each predictor *after taking account of the effect of all other predictors in the model.*
- ε_t is a white noise error term

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NOT RECOMMENDED!

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Seasonal dummies

- For quarterly data: use 3 dummies
- For monthly data: use 11 dummies
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Public holidays

- For daily data: if it is a public holiday, dummy=1, otherwise dummy=0.

Holidays

For monthly data

- Christmas: always in December so part of monthly seasonal effect
- Easter: use a dummy variable $v_t = 1$ if any part of Easter is in that month, $v_t = 0$ otherwise.
- Ramadan and Chinese New Year similar.

Distributed lags

Lagged values of a predictor.

Example: x is advertising which has a delayed effect

$$
\begin{aligned}
x_1 &= \text{advertising for previous month;} \\
x_2 &= \text{advertising for two months previously;} \\
&\vdots \\
x_m &= \text{advertising for } m \text{ months previously.}
\end{aligned}
$$

Fourier series

Periodic seasonality can be handled using pairs of Fourier terms:

$$s_k(t) = \sin\left(\frac{2\pi kt}{m}\right) \qquad c_k(t) = \cos\left(\frac{2\pi kt}{m}\right)$$

$$y_t = a + bt + \sum_{k=1}^{K}\left[\alpha_k s_k(t) + \beta_k c_k(t)\right] + \varepsilon_t$$

- Every periodic function can be approximated by sums of sin and cos terms for large enough K.
- Choose K by minimizing AICc or CV.
- Called “harmonic regression”

Your turn

1. Fit a regression model with a piecewise linear trend with Fourier terms for the US leisure employment data.

```
leisure <- us_employment |>
  filter(
    Title == "Leisure and Hospitality",
    year(Month) > 2001
  ) |>
  mutate(Employed = Employed / 1000) |>
  select(Month, Employed)
```

2. Does the model fit well? What are the implications for forecasting?

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Maximizing $\bar{R}^2$ is equivalent to minimizing $\hat{\sigma}^2$.

$$\hat{\sigma}^2 = \frac{1}{T-k-1}\sum_{t=1}^{T}\varepsilon_t^2$$

Akaike's Information Criterion

$$\text{AIC} = -2\log(L) + 2(k+2)$$

- L = likelihood
- k = # predictors in model.
- AIC penalizes terms more heavily than $\bar{R}^2$.

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$$\text{AIC}_\text{C} = \text{AIC} + \frac{2(k+2)(k+3)}{T-k-3}$$

- Minimizing the AIC or AICc is asymptotically equivalent to minimizing MSE via **leave-one-out cross-validation** (for any linear regression).

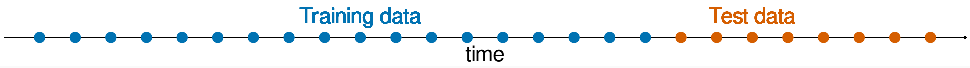
Leave-one-out cross-validation

For regression, leave-one-out cross-validation is faster and more efficient than time-series cross-validation.

- Select one observation for test set, and use *remaining* observations in training set. Compute error on test observation.
- Repeat using each possible observation as the test set.
- Compute accuracy measure over all errors.

Cross-validation

Traditional evaluation



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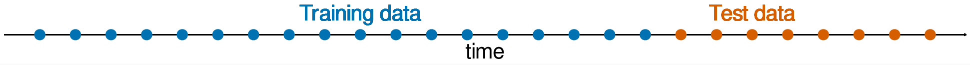
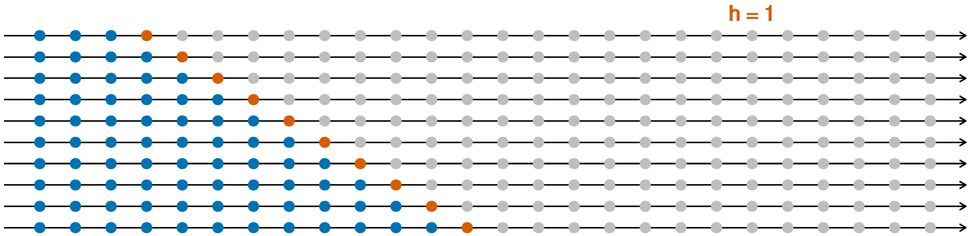


Time series cross-validation



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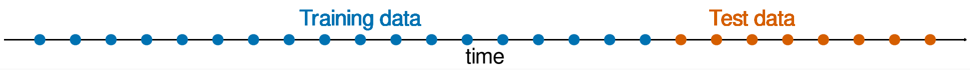
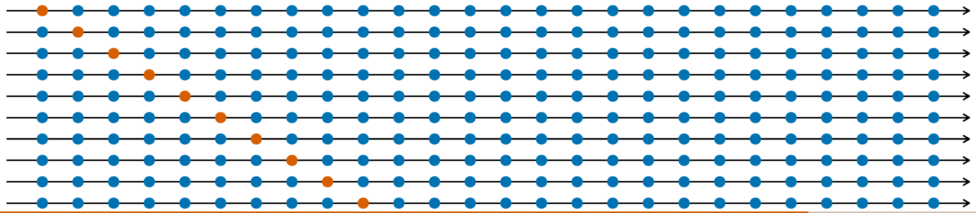


Leave-one-out cross-validation



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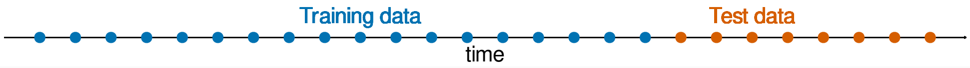

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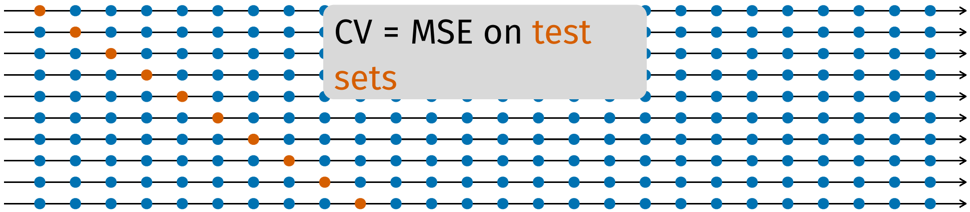

CV = MSE on test sets

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where L is the likelihood and k is the number of predictors in the model.

- BIC penalizes terms more heavily than AIC
- Also called SBIC and SC.
- Minimizing BIC is asymptotically equivalent to leave-v-out cross-validation when $v = T[1 - 1/(log(T) - 1)]$.

Choosing regression variables

Best subsets regression

- Fit all possible regression models using one or more of the predictors.
- Choose the best model based on one of the measures of predictive ability (CV, AIC, AICc).

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Backwards stepwise regression

- Start with a model containing all variables.
- Subtract one variable at a time. Keep model if lower CV.
- Iterate until no further improvement.
- Not guaranteed to lead to best model.

Ex-ante versus ex-post forecasts

- *Ex ante forecasts* are made using only information available in advance.
  - require forecasts of predictors
- *Ex post forecasts* are made using later information on the predictors.
  - useful for studying behaviour of forecasting models.
- trend, seasonal and calendar variables are all known in advance, so these don't need to be forecast.

Your turn

3. Produce forecasts of US leisure employment using your best regression model.
4. Why don’t you need to forecast the predictors?